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Financial Complexity and Trade
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Abstract

What are the implications on trading activity if investors are not sophisticated enough to understand and evaluate trades that have a complex payoff structure? Can frictions generated by this type of financial complexity be so severe that they lead to a complete market freeze, like that of the recent financial crisis? We show that for smooth convex preferences, including subjective expected utility, even extreme complexity cannot halt trade, unlike what happens for non-smooth preferences, such as maxmin expected utility. In the latter case, policies that make complex securities easier to understand or investors more sophisticated have a positive welfare effect, as they allow for existing gains from trade to materialise.

JEL-Classifications: D70, G01.

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1 Introduction

An implicit assumption when modelling financial markets is that each investor is sophisticated enough to be able to understand and trade any available security, however complex it might be. In reality, however, cognitive limitations do exist. Investors

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may have limited attention and time, be unaware of certain dimensions of the payoff structure, have difficulty formulating complex plans or lack special training. Moreover, information acquisition about past performance of some securities may be too costly. The Economic Affairs Committee (2009) reports that "It is hard for investors to evaluate complex financial instruments, because difficult risk modelling is required, and because they are often unaware of the details of the asset pool which backs financial securitisations".

To provide an example, consider the following description of security Jayanne 4, which was marketed by Credit Agricole in 2007 (Célérier and Vallée (2016)):

> This is a growth product linked to a basket composed of the FTSE Euro First 80, the FTSE 100, the SMI and the NIKKEI 225. The Annual Performance is set at 5% for the first three years. In the following years, if the performance since the start date of the worst-performing index is positive or null, then the Annual Performance for that year is registered at 5%, otherwise 0%. The Basket Performance since the start date is registered every six months. The Final Basket Performance is calculated as the average of all these six-monthly readings, capped at a maximum basket performance of 100%. After 8 years, the product offers a guaranteed capital return of 100%, plus the greater of either the sum of the Annual Performances, or 100% of the Final Basket performance.

A typical investor probably understands the indices FTSE Euro First 80, FTSE 100, SMI and NIKKEI 225, which are the basic ingredients of the security's payoff structure. In other words, he understands a "simple" bet that pays 1 if FTSE 100 is above 6000 tomorrow and 0 otherwise. However, he may fail to understand the "complex" Jayanne 4, even though it is "just" a combination of these four indices.

Another example of a complex security is the Collaterized Debt Obligation (CDO), which pools together cash-flow generating assets (mortgages, bonds and loans) and repackages them into discrete tranches. Because each tranche has a different risk profile and usually incorporates hundreds of thousands of underlying assets, it is a complicated task to work out its payoff structure, even though it is easy to understand the payoff structure of each separate underlying asset.

A result of these cognitive limitations is that although investors may be able to trade "simple" securities, they may fail to consider all of their possible combinations when formulating their portfolio. Polkovnichenko (2005) reports data from the Survey of Consumer Finances, showing that many households invest significant fractions of their wealth simultaneously in well-diversified mutual funds and in un-diversified

portfolios of individual stocks. Nieuwerburgh and Veldkamp (2010) derive optimal under-diversification in a framework with costly information acquisition. Carlin et al. (2013) show experimentally that complexity makes subjects less inclined to trade.

Is it possible that the effect of these limitations is so large that opportunities for trade cease to exist completely? We study this question in a complete markets environment with general convex preferences, where all Arrow-Debreu (A-D) securities are available (paying 1 if some state occurs and 0 otherwise).

The complexity of a trade is defined to be the minimum number of different A-D securities that can generate it. A trade that provides a different payoff at every state is generated by a combination of all A-D securities, hence its complexity is n, the number of states. A trade whose payoff differs only with respect to whether a state has occurred or not is generated by the respective A-D security, hence its complexity is 1. Cognitively constrained investors cannot formulate trades that are complex, even if their welfare would increase as a result.

Our notion of complexity effectively allows investors to formulate only "coarse" trades, which are measurable with respect to a coarse partition of the state space. Maximum perceived complexity then only allows coarse trades that are measurable with respect to a two-element partition, consisting of a state and its complement. Gul et al. (2017) study investors with cognitive limitations who, as a result, formulate only "coarse" (final) consumption plans, even though they can finance them using all available trades, even complex ones. We elaborate on the differences of these two approaches in the next section. However, it is important to emphasise that maximum perceived complexity is very different (but less restrictive) from a standard incomplete markets structure with one available A-D security. The former allows investors to buy or sell *any* A-D security but not combine two or more, whereas the latter allows them to trade only the unique security that is available.

Holding preferences and the initial allocation fixed, if there is trade without perceived complexity, so that there are gains from trade, will there always be trade with maximum perceived complexity? We analyse this question by characterising the existence of trading in terms of the investors' beliefs (Theorem 2) and by refining the notion of Pareto dominance in environments with uncertainty and diverse beliefs (Proposition 1).

The answer depends on how investors perceive risk and uncertainty. In particular, we find that for smooth preferences, which do not allow for kinked indifference curves (e.g. subjective expected utility, smooth ambiguity, multiplier and mean-variance preferences), trading will not stop because of maximum perceived complexity. This is a robust result, because it implies that trade will not stop also for all less extreme defini-

tions of complexity, where investors are able to trade more than one simple security at once, formulating less coarse trades. However, we also find that complexity can completely shut down trade if preferences are non-smooth, so that the indifference curve of at least one investor has a kink at his initial allocation. We provide such an example in Section 2 with maxmin expected utility (MEU) and a full insurance allocation.

This dichotomy of preference models provides behavioral implications. Suppose that there is trading in an environment where we control the perceived complexity, which is the maximum number k of different A-D securities that each investor can use to formulate a trade. If we observe that trading stops as k decreases, so that perceived complexity increases, it must be that some of the investors' preferences are non-smooth.

More importantly, if this type of perceived financial complexity has the potential to freeze trade when gains from trade exist, it could be argued that there is room for policy intervention. Brunnermeier and Oehmke (2010) argue that identifying complex securities is important in establishing whether they should be regulated subject to an FDA-type approval system, or limit who is able to invest in them. This paper provides an argument for supporting such an intervention. The example of Section 2 shows that, because of perceived complexity, there is no trade, however there are gains from trade that could materialize if investors were more sophisticated. This is a case where welfare will increase by policy intervention which forces those who market securities to make them simpler to understand, or increases the financial literacy of potential investors.

The model can also provide an explanation of why trading froze in some markets during the recent financial crisis. Suppose that financial crises generate a lot of uncertainty (Caballero and Simsek (2013), Brunnermeier and Sannikov (2014)) and that investors have non-smooth preferences. If we know that during the crisis the investors' priors about fundamentals do not change significantly but we nevertheless observe a market freeze, this can be interpreted as the investors perceiving the asset structure of this market to be complex.

To provide an example, Acharya et al. (2009) describe how a series of events that was triggered by an unexpected decrease of the US house prices in the first quarter of 2006 led to the freezing of the market for asset-backed commercial paper in 2007, right after BNP Paribas announced that it was suspending redemptions from its structured investment vehicles, which were trading these types of securities. We can interpret the once in a lifetime decline of US house prices as an event that created uncertainty about fundamentals. However, non-smooth preferences were not sufficient for shutting down trade. This happened one year later, exactly when the suspension of redemptions informed everyone that asset-backed commercial paper was no longer easy to price and

value. That this second event, which did not convey any information about fundamentals, triggered an immediate suspension of trade, could be explained by investors realising that these securities have a complex payoff structure and therefore cannot simultaneously trade any subset of them.

1.1   Relation to literature

Our paper is related to Billot et al. (2000, 2002), who characterize trading, from a full insurance allocation, in terms of disjoint sets of priors in a complete markets setting with a full insurance allocation, using the MEU and the Choquet expected utility (CEU) models. Rigotti et al. (2008) generalize these results for all convex preferences, encompassing many models with ambiguity averse preferences. Additionally, they characterize trading from any initial allocation at a (not necessarily full insurance) allocation. Ghirardato and Siniscalchi (2015) analyze the case of non-convex preferences.

In the MEU model with two investors and a full insurance allocation, Kajii and Ui (2006) characterize trading in the case of a partition with two elements, which in our terminology is an incomplete markets structure with only one security and maximum perceived complexity. In the special case where each investor's set of priors is the core of a convex capacity, they also characterize trade in an economy with more than one security and effectively show that trade in the absence of perceived complexity is equivalent to trade in the presence of maximum perceived complexity. Dominiak et al. (2012) extend this result for the CEU model with not necessarily convex capacities. We improve on these results with Theorem 2, by providing a characterization for any finite number of investors with general convex preferences and any initial allocation, as well as for asset structures with more than one security. More importantly, our characterization applies irrespective of whether trade in the absence of complexity is equivalent to trade in the presence of maximum perceived complexity, which is crucial in separating the models in terms of whether they satisfy this property.

In order to derive the characterization we additionally impose a no arbitrage condition, which has clear economic content and is related to the literature on how to refine the notion of Pareto dominance in environments with uncertainty and diverse beliefs. Gilboa et al. (2014) and Brunnermeier et al. (2014), among others, argue that Pareto dominance is not as compelling when there is uncertainty and investors have different priors in a Savage framework. Gilboa et al. (2014) propose a refinement of Pareto dominance which specifies that investors would still be better off under a common prior, whereas Brunnermeier et al. (2014) require this to occur for all beliefs in

the convex hull of the investors' beliefs. Our no arbitrage condition applies to general convex preferences, not just subjective expected utility, nevertheless it has a similar flavour because it restricts the investors' subjective beliefs to be not too different as to allow for an arbitrage trade.

Gul et al. (2017) study investors with cognitive limitations that have coarse (final) consumption plans, instead of coarse trades, as in the present model. For example, maximum perceived complexity in their model means that the investor chooses among all consumption plans that are measurable with respect to a two-element partition of the state space. However, to finance these consumption plans he is allowed to trade any combination of the A-D securities, thus generating a complex payoff structure, according to our terminology. In contrast, in our model the investor can only formulate a trade that is measurable with respect to a two-element partition. However, if his initial endowment is measurable with respect to the finest partition, then so is his final consumption plan. See Section 5.3 in Gul et al. (2017) for a comparison of the two approaches. Moreover, their focus is different from ours, as they show that allocations are riskier and prices are more extreme when compared to the no perceived complexity case, whereas we examine whether trade would occur.

Our definition of complexity relates to how investors perceive the payoff structure. Alternatively, Caballero and Simsek (2013) use ambiguity and the notion of complexity about the structure of cross exposures of banks to explain market freezes.

Rigotti and Shannon (2005) characterize Pareto optima and equilibria in the incomplete preferences model of Knightian uncertainty of Bewley (1986). This model is used also by Easley and O'Hara (2010) to explain no trade. Rigotti and Shannon (2012) show that generic determinacy is a robust feature of general equilibrium models with ambiguity averse preferences, because kinks are relatively rare, whereas robust indeterminacies arise naturally in the model of Bewley (1986), where kinks are ubiquitous. Although we prove our results only for complete convex preferences, it is straightforward to extend them in the incomplete preferences model of Bewley (1986).[1] In this model, the indifference curve at any endowment has a kink, hence maximum complexity would generically shut down trade, as opposed to some models with ambiguity aversion, where indifference curves are smooth at non full insurance endowments.

Our main difference from these papers is that we use complexity in order to explain no trade in an environment where there are actually gains from trade. This difference has policy implications, because we suggest that if complexity was lifted then there would be gains from trade, whereas the aforementioned papers suggest that (in the case

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[1]See Section 4 for details.

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of no trade) uncertainty has destroyed all gains from trade. Moreover, our mechanism can help explain the BNP Paribas story, where the market froze not because of the initial event that created uncertainty (and hence ambiguity or incompleteness) about house prices, but due to the investors' realization that the payoff structure was too difficult to understand.

Rigotti and Shannon (2005) provide conditions under which endogenous incomplete markets can arise. Roughly, if investors have different but precise probabilities about some states but similar but imprecise probabilities about the remaining states, then they trade only A-D securities contingent on the former set, so it is as if the latter securities are missing. Our focus is different, because we ask whether trade would still occur in the case of maximum perceived complexity, which is not the same as an incomplete markets structure with one available security. Since investors have different and precise probabilities about at least one state, maximum perceived complexity does not shut down trade in any such setting with endogenous incomplete markets.

Lang (2017) defines first-order and second-order ambiguity aversion and characterizes them in term of whether the indifference curve at the endowment point has a kink or it is smooth. He also provides several economic examples where this dichotomy matters. Using his terminology, the present paper shows that second-order ambiguity aversion implies that maximum perceived complexity does not shut down trade. Mihm (2016) proposes a model of reference-dependent MEU preferences where the indifference curve has a kink at the endowment, so that using our results maximum perceived complexity can shut down trade.

Our paper is related to the growing literature on complex securities (Amromin et al. (2011), Henderson and Pearson (2011), Valkanov and Ghent (2014), Griffin et al. (2014), Hens and Rieger (2014), Sato (2014)). Célérier and Vallée (2016) study more than 50,000 securities and show that complexity has increased over time. Simsek (2013) shows that complexity increases opportunities for speculation in a model with heterogeneous beliefs. He uses smooth (mean-variance) preferences for which, according to the present paper, complexity does not shut down trade.

Our notion of complexity specifies that the investor has a coarse understanding of his available trading strategies. Alternatively, several strands of the literature study the coarse understanding of the state space, such as in decision theory (Dekel et al. (2001), Epstein et al. (2007), Ahn and Ergin (2010)), unawareness (Fagin and Halpern (1988), Heifetz et al. (2006), Galanis (2013)) and inattention (Sims (2003), Woodford (2012), Gabaix (2014)).

The paper is organized as follows. In the next section we provide an example which illustrates our approach. Section 3 introduces the model and characterizes the

occurrence of trade in the case of maximum perceived complexity. Section 4 concludes.

2 Illustration

We illustrate our approach using an example. Let $S = \{s_1, s_2, s_3\}$ be the state space, describing the uncertainty about tomorrow. Consider a standard complete markets setting with three A-D securities, each paying 1 if a particular state $s \in S$ occurs and 0 otherwise. The economy consists of two investors, i and j, who have the same endowment $e = (5, 5, 5)$, paying 5 at every state. We call this a full insurance allocation. Their preferences are represented by maxmin expected utility (MEU) with $u^i(x) = x$, $x \in \mathbb{R}_+$. In particular, i's utility from trade $f^i \in \mathbb{R}^3$, where $e + f^i \geq 0$, is

$$V^i(e + f^i) = \min_{q \in C^i} \sum_{s \in S} u^i(e(s) + f^i(s)) q(s),$$

where $C^i \subseteq \Delta S$ is a compact and convex set of beliefs.

Suppose that C^i is the convex hull of probabilities $p_1 = (0.2, 0.6, 0.2)$, $p_2 = (0.2, 0.4, 0.4)$ and $p_3 = (0.3, 0.5, 0.2)$, whereas C^j is the convex hull of probabilities $q_1 = (0.4, 0.4, 0.2)$ and $q_2 = (0.3, 0.4, 0.3)$.
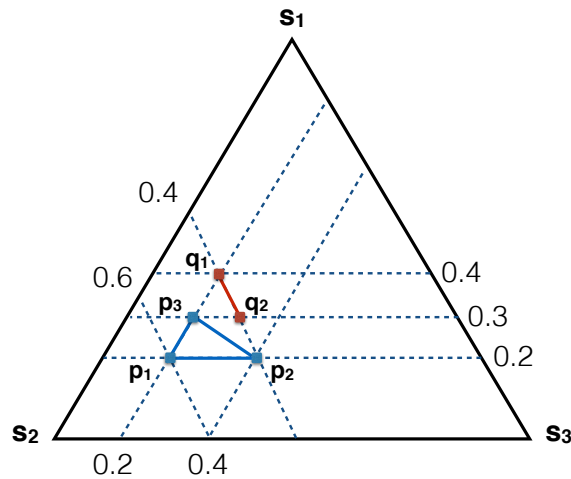

Figure 1: Trade occurs only in the absence of complexity

These sets are shown in Figure 1. The triangle represents the probability simplex, so that each point represents a probability on $\{s_1, s_2, s_3\}$. A dashed line, which is

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parallel to a side of the triangle, contains all probabilities that assign the same weight on the state depicted opposite to that side. Set C^i is the triangle formed by p_1, p_2 and p_3, whereas C^j is the line formed by q_1 and q_2.

Because C^i and C^j are disjoint, there are trades that will make both strictly better off than consuming their endowment, which gives utility 5. For example, consider $f^i = (-3.2, 2.5, -0.5)$ and $f^j = (3.2, -2.5, 0.5)$. This is a trade because $f^i + f^j = 0$. Then, $V^i(e + f^i) = 5.16$ and $V^j(e + f^j) = 5.11$. Billot et al. (2000) show that in the MEU model with a full insurance allocation, there is a trade (i.e. the initial allocation is not Pareto optimal) if and only if the sets of beliefs are disjoint.

According to our terminology, $\{f^i, f^j\}$ is a complex trade, because it provides a different payoff at every state, hence its construction requires a combination of all three A-D securities.

Suppose now that each investor is cognitively constrained, so that he can buy or sell at most one A-D security and cannot combine two or more to construct a complex trade. In other words, he can only formulate a coarse trade, which is measurable to a two-element partition of S, consisting of a state and its complement. Then, trade between the two investors translates to i betting on state s occurring, hence buying the A-D security, and j betting on s not occurring, hence selling the same security.

Consider an A-D security on state s_1, with price $c < 1$. If investor i sells it, he bets that s_1 will not occur and the trade he gets is $f^i = (c - 1, c, c)$, whereas if j buys it he gets $f^j = (1 - c, -c, -c)$.

Let $\overline{p}^i(s) = \max_{p \in C^i} p(s)$ and $\underline{p}^i(s) = \min_{p \in C^i} p(s)$ be i's maximum and minimum belief on state s. If both investors agree on this simple trade $\{f^i, f^j\}$, then investor i must strictly prefer $e + f^i$ over e, and similarly for j. In particular, $V^i(e + f^i) = 5 + \overline{p}^i(s_1)(c - 1) + (1 - \overline{p}^i(s_1))c > V^i(e) = 5$ and $V^j(e + f^j) = 5 + \underline{p}^j(s_1)(1 - c) - (1 - \underline{p}^j(s_1))c > V^j(e) = 5$. These inequalities imply $\underline{p}^j(s_1) > c > \overline{p}^i(s_1)$. In other words, i agrees to sell if all of his beliefs place a small weight on s_1 happening, relative to j's beliefs. Similarly, i buys the A-D security on s_1 that j sells if $\overline{p}^j(s_1) < c < \underline{p}^i(s_1)$. Put more compactly, if the two investors agree to bet on s_1 then $[\underline{p}^i(s_1), \overline{p}^i(s_1)] \cap [\underline{p}^j(s_1), \overline{p}^j(s_1)] = \emptyset$ and the converse is also true.

In this example the two investors do not agree to bet on s_1 because $[\underline{p}^i(s_1), \overline{p}^i(s_1)] \cap [\underline{p}^j(s_1), \overline{p}^j(s_1)] \neq \emptyset$, and the same is true for s_2 and s_3. Can we generalise this result, so that we find sets of beliefs, $\overline{C}^i$ and $\overline{C}^j$, such that they are disjoint if and only if the two investors agree to bet on *some* state or, equivalently, to trade a particular A-D security?

Let $\overline{C}^i$ be the set of probability measures p such that $\underline{p}^i(s) \leq p(s) \leq \overline{p}^i(s)$ for all

states s, and similarly for $\overline{C}^j$. In Figure 1, $\overline{C}^j = C^j$ is still the convex hull of q_1 and q_2. However, $\overline{C}^i$ is bigger than C^i, as it is the convex hull of p_1, p_2, p_3 and q_2. It is constructed by including all probabilities that are within the respective dashed lines.

It is straightforward that if $[\underline{p}^i(s), \overline{p}^i(s)] \cap [\underline{p}^j(s), \overline{p}^j(s)] = \emptyset$ for some s (so that there is betting on that state), then $\overline{C}^i$ and $\overline{C}^j$ are disjoint. But the converse is not true. A counterexample in Section 3.5 shows that even if $[\underline{p}^i(s), \overline{p}^i(s)] \cap [\underline{p}^j(s), \overline{p}^j(s)] \neq \emptyset$ for all states, so that there is never any betting, $\overline{C}^i$ and $\overline{C}^j$ may still be disjoint. However, it turns out that these cases are pathological, because they violate the following no arbitrage condition: it is not possible for any investor to successfully offer to others a series of bets that others will accept and it will give him a positive payoff at all states. Sets $\overline{C}^i$ and $\overline{C}^j$ have q_2 as a common element, which is consistent with no betting on any state.

The example shows that starting from an allocation where there are gains from trade and as the perceived complexity increases, it is as if the beliefs of the investors expand, leading to a result of no trade. This is of course very specific because it assumes MEU with linear u, two investors and a full insurance allocation. However, we show that the arguments can be substantially generalised to accommodate any allocation and any number of investors with preferences that are complete, transitive, strongly monotonic and convex.

Instead of starting from a set C^i of beliefs which are specific to the MEU model, we follow Rigotti et al. (2008) and consider the set of subjective beliefs $\pi^i(e^i)$, which are the prices of A-D securities, normalised to sum to 1, such that i would prefer not trading his endowment e^i.[2] Mathematically, subjective beliefs are the normals of the supporting hyperplanes of the indifference curve at the endowment point. If preferences are smooth at the endowment then $\pi^i(e^i)$ is a singleton, but if there is a kink then it is a general convex and compact set. In the standard case of no perceived financial complexity, Rigotti et al. (2008) show that an allocation is Pareto optimal if and only if there is a common subjective belief.

In the case of maximum perceived complexity, we establish that two investors agree to bet on a state if and only if the intervals defined by their maximum and minimum subjective beliefs on that state are disjoint. For this result, convexity of preferences is crucial. We then define $\overline{\pi}^i(e^i)$ in the same way as $\overline{C}^i$ and say that there is an $\overline{S}$-common belief if the intersection for all investors is nonempty. Our main result, Theorem 2, specifies that there is no $\overline{S}$-common belief if and only if there is trade in an environment with maximum perceived complexity. To achieve this characterization, we

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[2]In the MEU model with a full insurance allocation, $C^i = \pi^i(e^i)$.

assume a no arbitrage condition, which specifies that it is not possible for an investor to obtain a strictly positive payoff at each state by successfully offering a bet to all other investors. Proposition 1 characterizes this no arbitrage condition in terms of the investors' subjective beliefs.

It is straightforward that trade in the presence of maximum perceived complexity implies trade in the absence of perceived complexity. However, the reverse is not true, as shown in Figure 1, which means that as complexity increases (or investors become less sophisticated), trade may eventually stop completely. Nevertheless, trade in one environment is equivalent to trade in the other environment if each investor's set of subjective beliefs consists of a single probability measure, which is the case of smooth (differentiable) indifference curves at the endowment point. This observation allows us to obtain a dichotomy of models with convex preferences (including most models with ambiguity aversion), in terms of whether complexity impedes trade.

3 Model

3.1 Set up

Consider a set I of investors with typical element i and a single consumption good. Uncertainty is represented by a finite set of payoff relevant states S, with typical element s. The set of consequences is $\mathbb{R}_+$, interpreted as monetary payoffs. Investor i has convex preference relation $\succsim^i$ on the set of acts $\mathcal{F} = \mathbb{R}_+^S$, which satisfies the following standard axioms.

Axiom 1. *(Preference).* $\succsim^i$ *is complete and transitive.*

Axiom 2. *(Continuity). For all $f \in \mathcal{F}$, the sets $\{g \in \mathcal{F} : g \succsim^i f\}$ and $\{g \in \mathcal{F} : f \succsim^i g\}$ are closed.*

Axiom 3. *(Convexity). For all $f \in \mathcal{F}$, the set $\{g \in \mathcal{F} | g \succsim^i f\}$ is convex.*

Axiom 4. *(Strong Monotonicity). For all $f \neq g$, if $f \geq g$, then $f \succ^i g$.*

An economy is a tuple $\{\succsim^i, e^i\}_{i \in I}$, where $|I| \geq |S| + 1$ and $\{e^i\}_{i \in I} \in \mathbb{R}_{++}^{SI}$ is the interior initial allocation. An economy is large if there are at least $|S|$ copies for each preference relation $\succsim^i$.

We assume a complete market with a collection $\{d_s\}_{s \in S}$ of A-D securities, where d_s has price p_s and pays 1 if state s occurs and 0 otherwise. A portfolio $\{a_s\}_{s \in S}$ at prices $\{p_s\}_{s \in S}$ generates net trade $f \in \mathbb{R}^S$ such that $f = \sum_{s \in S} a_s d_s - \mathbf{1} \sum_{s \in S} a_s p_s$, where

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a_s denotes the units of security d_s which are bought if $a_s > 0$ or sold if $a_s < 0$ and $\mathbf{1}$ pays 1 at every state. Investor i weakly prefers this portfolio to his initial endowment if $e^i + f \succsim^i e^i$.

3.2 Subjective beliefs

Based on Yaari (1969), Rigotti et al. (2008) define investor i's *subjective beliefs at an act* f to be the normals (normalized to be probabilities) of all supporting hyperplanes at f,

$$\pi^i(f) = \{p \in \Delta S : \mathbb{E}_p g \geq \mathbb{E}_p f \text{ for all } g \succsim^i f\},$$

where $\mathbb{E}_p f = \sum_{s \in S} p(s) f(s)$ is the expectation of f given probability measure p. For convex preferences, $\pi^i(f)$ is nonempty, convex and compact.

Rigotti et al. (2008) establish the following two properties for strictly positive acts f and convex preferences.[3] First, $\mathbb{E}_p f \geq \mathbb{E}_p g$ for some $p \in \pi^i(f)$ implies $f \succsim^i g$. Second, $\mathbb{E}_p g > \mathbb{E}_p f$ for all $p \in \pi^i(f)$ implies $\epsilon g + (1 - \epsilon) f \succ^i f$ for sufficiently small $\epsilon > 0$.

3.3 Common beliefs

We say that there is a common belief at initial allocation $\{e^i\}_{i \in I}$ if $\bigcap_{i \in I} \pi^i(e^i) \neq \emptyset$. For each $s \in S$, let $\underline{p}^i(s) = \min_{p \in \pi^i(e^i)} p(s)$ and $\overline{p}^i(s) = \max_{p \in \pi^i(e^i)} p(s)$ be i's minimum and maximum subjective belief about s, respectively. Let

$$\overline{\pi}^i(e^i) = \{q \in \Delta S : \underline{p}^i(s) \leq q(s) \leq \overline{p}^i(s) \text{ for all } s \in S\}$$

be the set of probability measures that are within i's minimum and maximum subjective beliefs at e^i, for each $s \in S$.[4] We next define a weaker notion of common beliefs.

Definition 1. *There is an $\overline{S}$-common belief at $\{e^i\}_{i \in I}$ if $\bigcap_{i \in I} \overline{\pi}^i(e^i) \neq \emptyset$.*

Because $\pi^i(e^i) \subseteq \overline{\pi}^i(e^i)$, if there is a common belief then there is a $\overline{S}$-common belief but the converse is not true, as shown in Figure 1. However, if $\pi^i(e^i)$ is a singleton for each i, then the two notions are equivalent.[5] In Section 3.7, we show that there is no $\overline{S}$-common belief if and only if there is trade in the presence of maximum perceived complexity.

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[3] An act f is strictly positive if $f(s) > 0$ for all $s \in S$.

[4] Note that $\overline{\pi}^i(e^i)$ is a closed and convex polytope, as it is bounded and the intersection of half spaces.

[5] The converse is not true, so that if the set of common beliefs is equal to the set of $\overline{S}$-common beliefs, it is not the case that each $\pi^i(e^i)$ is a singleton.

3.4   Trades and bets

We say that $f \in \mathbb{R}^S$ is a bet on state $s \in S$ if there exist $a, b \in \mathbb{R}$, $ab < 0$, such that $f(s') = a$ if $s' = s$ and $f(s') = b$ otherwise. A bet on s can be constructed by buying or short selling some units of an A-D security d_s that pays 1 if s occurs and 0 otherwise. A bet on s where $a > 0 > b$ can be generated by buying, at price $-\frac{b}{a-b}$, $a - b$ units of d_s. If state s does not occur, then the payoff is $(a - b)\frac{b}{a-b} = b < 0$. If s occurs, the payoff is $(a - b)\frac{b}{a-b} + a - b = a > 0$.

Similarly, a bet on s where $b > 0 > a$ can be generated by selling, at price $\frac{b}{b-a}$, $b - a$ units of d_s. If state s does not occur, then the payoff is $(b - a)\frac{b}{b-a} = b > 0$. If s occurs, the payoff is $(b - a)\frac{b}{b-a} - (b - a) = a < 0$.

Tuple $\{f^i\}_{i \in I} \in \mathbb{R}^{SI}$ is a *trade* if $\sum_{i \in I} f^i = 0$ and $e^i + f^i \geq 0$ for all $i \in I$. It is a *bet* if, additionally, each f^i is a bet on some s. Trade $\{f^i\}_{i \in I}$ is *agreeable* if $e^i + f^i \succ^i e^i$ for all $i \in I$. A trade is an *agreeable bet* if it is an agreeable trade and a bet.

3.5   No arbitrage

Gains from trade exist at an allocation if this allocation is not Pareto optimal. However, in an environment with uncertainty, the notion of Pareto dominance is not as compelling as in an environment with certainty. See Gilboa et al. (2014), Brunnermeier et al. (2014) and references therein for a discussion using the subjective expected utility model. In essence, these authors refine Pareto dominance by specifying conditions which ensure that the investors' beliefs are not too far apart. In this paper we refine Pareto dominance by imposing a no arbitrage condition, which restricts how far apart the investors' subjective beliefs can be, as shown further below by Proposition 1.

Definition 2. *Tuple $\{f^i\}_{i \in I} \in \mathbb{R}^{SI}$ is an arbitrage trade if it is an agreeable trade and there exist partitions $\{A, B\}$ of I and $\{S_i\}_{i \in B}$ of S such that:*

- *for all $i \in A$, $f^i(s) = k_i > 0$ for all $s \in S$,*

- *for all $i \in B$, $f^i = \sum_{s \in S_i} h_s$, where h_s is a bet on s,*

- *for all $i \in I$ and all $s \in S$, $e^i \succsim^i e^i - h_s$.*

It is an arbitrage bet if, additionally, each S_i is a singleton.

In an arbitrage trade there are two types of investors. Each $i \in A$ is an arbitrageur, receiving a positive and fixed payoff at each state. Each $i \in B$ is a bettor, willing to bet on some states. Moreover, no investor is willing to take the opposite side of bet h_s. In an arbitrage bet, each bettor bets only on one state.

Definition 3. *There is no arbitrage if there are no arbitrage trades, or there are no arbitrage bets and the economy is large.*[6]

The requirement of no arbitrage bets is weaker than the requirement of no arbitrage trades, however it is sufficient for our purposes when the economy is large. In particular, we use the notion of no arbitrage in the environment of maximum perceived complexity, in order to exclude cases where there is no bet on any state, yet there is no $\overline{S}$-common belief.

As we argued in the example of Section 2 and Theorem 2 below generalises, there is an agreeable bet on state s between i and j if and only if $[\underline{p}^i(s), \overline{p}^i(s)] \bigcap [\underline{p}^j(s), \overline{p}^j(s)] = \emptyset$, implying that either $\underline{p}^i(s) > \overline{p}^j(s)$ or $\underline{p}^j(s) > \overline{p}^i(s)$. In other words, the A-D prices about s that would sustain zero net demand are very different for i and j. More generally, define

$$[\underline{q}(s), \overline{q}(s)] \equiv \bigcap_{i \in I} [\underline{p}^i(s), \overline{p}^i(s)]$$

to be the conjunction of all the constraints about state s that an $\overline{S}$-common belief must satisfy. Then, $[\underline{q}(s), \overline{q}(s)] = \emptyset$ is equivalent to the existence of a bet on s between two investors.

If $[\underline{q}(s), \overline{q}(s)] = \emptyset$ for some state s, then it is straightforward that there is no $\overline{S}$-common belief. However, the converse is not true. It is possible that $[\underline{q}(s), \overline{q}(s)] \neq \emptyset$ for all $s \in S$, so that there is no agreeable bet on any state, yet there is no $\overline{S}$-common belief. However, in that case there are arbitrage opportunities.

To show this, consider the following example with four states and five investors. Investors $i = 1, 2$ have identical preferences and endowments. Their set of subjective beliefs $\pi^i(e^i)$ is the convex hull of the following three probabilities,

$$(0.2315, 0.0385, 0.2773, 0.4527),$$
$$(0.2306, 0.1668, 0.3948, 0.2078),$$
$$(0.1549, 0.0163, 0.4365, 0.3923).$$

Investors $i = 3, 4, 5$ have identical preferences and endowments. Their set of subjective beliefs $\pi^i(e^i)$ is the convex hull of probabilities

$$(0.0303, 0.5476, 0.3179, 0.1042),$$
$$(0.4765, 0.1215, 0.1492, 0.2528),$$
$$(0.3107, 0.2340, 0.3140, 0.1413).$$

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[6]Recall that an economy is large if each investor i has $|S|$ copies.

We then have that $[\underline{q}(s_1), \overline{q}(s_1)] = [0.1549, 0.2315]$, $[\underline{q}(s_2), \overline{q}(s_2)] = [0.1215, 0.1668]$, $[\underline{q}(s_3), \overline{q}(s_3)] = [0.2773, 0.3179]$ and $[\underline{q}(s_4), \overline{q}(s_4)] = [0.2078, 0.2528]$. There is no $\overline{S}$-common belief because any p that satisfies the previous four constraints has at most

$$\sum_{s \in S} p(s) \leq \sum_{s \in S} \overline{q}(s) = 0.969 < 1.$$

However, this example is problematic because it allows for arbitrage. Fix $\sum_{s \in S} \overline{q}(s) < 1$ and consider, for each state s, a bet f_{a_s} for investor i with endowment e^i and $\overline{q}(s) = \overline{p}^i(s)$, that pays $a_s - 1$ if s occurs and a_s otherwise, where a_s is bigger but arbitrarily close to $\overline{p}^i(s)$, so that $\sum_{s \in S} a_s < 1$. This bet can be generated by short selling an A-D security d_s at price a_s.

Because the expectation $\mathbb{E}_p(f_{a_s} + e^i) = a_s - p(s) + \mathbb{E}_p e^i > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$, convexity of preferences implies that for small enough $k > 0$, investor i strictly prefers the convex combination $k(f_{a_s} + e^i) + (1 - k)e^i = kf_{a_s} + e^i$ to e^i.

Investor 5 can offer bet $kf_{a_{s_1}}$ to investor 1, $kf_{a_{s_2}}$ to investor 2, $kf_{a_{s_3}}$ to investor 3 and $kf_{a_{s_4}}$ to investor 4, such that $\sum_{s \in S} a_s < 1$. Essentially, investor 5 is offering to buy k units of A-D security d_s at price a_s, for each state s. These bets are going to be accepted, because they make each $i = 1, 2, 3, 4$ strictly better off. Moreover, no other investor is willing to take the opposite side of each bet, $-kf_{a_s}$. However, investor 5's payoff at any state s is $-k \sum_{s' \neq s} a_{s'} - k(a_s - 1) = -k(\sum_{s \in S} a_s - 1) = k(1 - \sum_{s \in S} a_s) > 0$. Hence, all conditions of Definition 2 are satisfied.

The following Proposition generalises this result.

Proposition 1. *Suppose that for each $s \in S$, $[\underline{q}(s), \overline{q}(s)] \equiv \bigcap_{i \in I}[\underline{p}^i(s), \overline{p}^i(s)] \neq \emptyset$. Then, no arbitrage implies $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$. Conversely, $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$ implies that there are no arbitrage bets.*

Proof. Suppose that for each $s \in S$, $[\underline{q}(s), \overline{q}(s)] = \bigcap_{i \in I}[\underline{p}^i(s), \overline{p}^i(s)] \neq \emptyset$ and $\sum_{s \in S} \overline{q}(s) < 1$. Choose $a_s > \overline{q}(s)$ such that $\sum_{s \in S} a_s < 1$. We will construct an arbitrage trade $\{f^i\}_{i \in I}$, or an arbitrage bet in the case that the economy is large. Note that for each $s \in S$, $\overline{q}(s) = \overline{p}^i(s)$ for some $i \in I$ and $\overline{q}(s) \leq \overline{p}^j(s)$ for all $j \in I$.

Consider bet $f_{s,a}$ that pays $a - 1$ at s and a otherwise. For all a such that $a > \overline{p}^i(s) = \overline{q}(s)$, we have that $\mathbb{E}_p(f_{s,a} + e^i) = a - p(s) + \mathbb{E}_p e^i > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$. From the second property of π^i, there exists small enough $k \in (0, 1)$ such that $k(f_{s,a} + e^i) + (1 - k)e^i = kf_{s,a} + e^i \succ^i e^i$. Therefore, investor i would strictly prefer to get bet $kf_{s,a}$ which pays $ka - k$ at s and ka otherwise. Moreover, this is also true for any $0 < k_0 < k$.

The third condition of an arbitrage trade is also satisfied because $[\underline{q}(s), \overline{q}(s)] \neq \emptyset$ and $a > \overline{p}^i(s) = \overline{q}(s)$ imply $a > \underline{p}^i(s)$ for all $i \in I$. This means that $\mathbb{E}_p(-f_{s,a} + e^i) = -a + \underline{p}^i(s) + \mathbb{E}_p e^i < \mathbb{E}_p e^i$ for some $p \in \pi^i(e^i)$, hence the first property of the subjective beliefs implies that $e^i \succsim^i e^i - f_{s,a}$. From convexity, we also have $e^i \succsim^i e^i - kf_{s,a}$.

By repeating the same argument for each $s \in S$, we can create a tuple $\{k_s f_{s,a_s}\}_{s \in S}$ of bets. If the economy is not large, there is the possibility that for the same investor i we have $\overline{p}^i(s) = \overline{q}(s), \overline{p}^i(s') = \overline{q}(s')$ and this can be true for more than two states. Then, the same arguments show that investor i would strictly prefer to get $k_s f_{s,a_s} + k_{s'} f_{s',a_{s'}}$. However, in this case i receives an f^i that provides different payoffs in s, s' and $S \setminus \{s, s'\}$, hence it is an arbitrage trade and not an arbitrage bet. If the economy is large, then there are $|S|$ investors with the same preferences as i, hence we can assign to each copy a bet on a different state.

By setting $k = \min_{s \in S} k_s$, the new tuple is $\{kf_{a_s}\}_{s \in S}$, where for each i with $\overline{q}(s) = \overline{p}^i(s)$ we have $f^i = kf_{a_s}$ and $kf_{a_s} + e^i \succ^i e^i$. Because there are at least $|S| + 1$ investors, we can assign one investor i^* to take the opposite side of $\{kf_{a_s}\}_{s \in S}$, which yields $\sum_{s \in S} -kf_{a_s}$ and pays $-k \sum_{s' \neq s} a_{s'} - k(a_s - 1) = k(1 - \sum_{s \in S} a_s) > 0$ at all states. For any j who has not been offered kf_{a_s} for some $s \in S$, we let $f^j(s) = k_j > 0$ for all $s \in S$, where k_j is arbitrarily small. From Axiom 4, each j would strictly prefer to accept f^j and this is affordable because $\sum_{s \in S} -kf_{a_s}$ is strictly positive at all states, so i^* would get a slightly lower payoff at each state. Hence, we have created an arbitrage trade, or an arbitrage bet in the case of a large economy. We can create a similar arbitrage opportunity if $\sum_{i \in I} \underline{q}(s) > 1$, with bets that pay $k - ka_s$ at s and $-ka_s$ otherwise, where $\underline{q}(s) > a_s$, for small enough $k > 0$.

Conversely, suppose $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$ but there is an arbitrage bet $\{f^i\}_{i \in I}$. By definition we have $\sum_{i \in B} f^i < 0$ and $f^i + e^i \succ^i e^i$ for all $i \in B$, where f^i is of the form $f^i(s^i) = a^i$ and $f^i(s) = b^i$ for $s \neq s^i$, $a^i \neq b^i$. From the first property of $\pi^i(e^i)$ and Axiom 4, we have that $\mathbb{E}_p(e^i + f^i) > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$, all $i \in I$.

We next show that $\mathbb{E}_p f^i > 0$ for all $p \in \overline{\pi}^i(e^i)$ and all $i \in I$. For all $i \in A$ this is obvious, as they receive a fixed positive payoff at all states. Fix $i \in B$ and let $s^i = s$. Note that $\overline{p}^i(s)a^i + \underline{p}^i(S \setminus s)b^i > 0$ and $\underline{p}^i(s)a^i + \overline{p}^i(S \setminus s)b^i > 0$, because $1 - \overline{p}^i(s) = \underline{p}^i(S \setminus s)$ and $1 - \underline{p}^i(s) = \overline{p}^i(S \setminus s)$, where $\underline{p}^i(E) = \min_{p \in \pi^i(e^i)} p(E)$ and $\overline{p}^i(E) = \max_{p \in \pi^i(e^i)} p(E)$, for any event $E \subseteq S$. Take $p \in \overline{\pi}^i(e^i)$. If $p(s) = \overline{p}(s)$ or $p(s) = \underline{p}(s)$ then we are done. Suppose $\underline{p}(s) < p(s) < \overline{p}(s)$ and let $q_1(s) = \underline{p}(s)$, $q_2(s) = \overline{p}(s)$, where $q_1, q_2 \in \pi^i(e^i)$. Let $\lambda \in (0, 1)$ such that $\lambda q_1(s) + (1 - \lambda)q_2(s) = p(s)$, which implies that

$\lambda q_1(S \setminus s) + (1 - \lambda)q_2(S \setminus s) = p(S \setminus s)$. We also have that $q_1(s)a^i + q_1(S \setminus s)b^i > 0$ and $q_2(s)a^i + q_2(S \setminus s)b^i > 0$. Multiplying with $\lambda, 1 - \lambda$, and adding the two inequalities we have that $\mathbb{E}_p f^i = p(s)a^i + p(S \setminus s)b^i > 0$.

Because $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$, there exists $a \in [0, 1]$ such that $a \sum_{s \in S} \underline{q}(s) + (1 - a) \sum_{s \in S} \overline{q}(s) = 1$. For each $s \in S$, let $p(s) = a\underline{q}(s) + (1-a)\overline{q}(s)$. We then have $\sum_{s \in S} p(s) = 1$, so that p is a probability measure such that $p(s) \in [\underline{q}(s), \overline{q}(s)]$ for all $s \in S$, hence $p \in \bigcap_{i \in I} \overline{\pi}^i(e^i) \neq \emptyset$. Using this p and by adding $\mathbb{E}_p f^i > 0$ over all $i \in B$ we have $\sum_{i \in B} \mathbb{E}_p f^i = \mathbb{E}_p \sum_{i \in B} f^i < 0$, a contradiction.

$\square$

3.6 Absence of complexity

Consider first the standard environment of no perceived complexity, where each investor can understand and trade any combination of the A-D securities. In other words, his trade can be measurable with respect to any partition of S, even the finest one. We say that there is trade in the absence of complexity if there is an agreeable trade $\{f^i\}_{i \in I} \in \mathbb{R}^{SI}$, so that $\{e^i\}_{i \in I}$ is not Pareto optimal. Formally, $\sum_{i \in I} f^i = 0$ and $e^i + f^i \succ^i e^i$ for all $i \in I$.[7] Proposition 7 in Rigotti et al. (2008) shows that interior allocation $\{e^i\}_{i \in I}$ is not Pareto optimal if and only there is no common belief, so that $\bigcap_{i \in I} \pi^i(e^i) = \emptyset$.[8]

Theorem 1. *There is trade in the absence of complexity if and only if there is no common belief.*

3.7 Maximum perceived complexity

Consider now a more restrictive setting, where preferences and the allocation $\{e^i\}_{i \in I}$ are still the same, however each investor is so unsophisticated or cognitively constraint that he can buy or sell at most one A-D security, instead of any linear combination, as in the previous section. This means that his available trades are coarse, as they need to be measurable with respect to a two-element partition $\{s, S \setminus s\}$ of the state space.

We say that there is trade in the presence of maximum perceived complexity if there is an agreeable bet. The main result of the paper characterises the existence of trading

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[7]Note that because of Axiom 4, $\{e^i\}_{i \in I}$ is not Pareto optimal ($\sum_{i \in I} f^i = 0$, $f^i + e^i \succsim^i e^i$ for all $i \in I$ and $f^j + e^j \succ^j e^j$ for some $j \in I$) if and only if $\sum_{i \in I} f^i = 0$ and $e^i + f^i \succ^i e^i$ for all $i \in I$.

[8]Theorem 3 in Rigotti and Shannon (2005) proves the same result in the incomplete preferences model of Bewley (1986).

in the presence of maximum perceived complexity, under a no arbitrage condition.

Theorem 2. *Under no arbitrage, there is trade in the presence of maximum perceived complexity if and only if there is no $\overline{S}$-common belief.*

Proof. Suppose that $\bigcap_{i \in I} \overline{\pi}^i(e^i) = \emptyset$. We first show that $\bigcap_{i \in I} [\underline{p}^i(s), \overline{p}^i(s)] = \emptyset$ for some $s \in S$. Suppose not, so that for each $s \in S$, $[\underline{q}(s), \overline{q}(s)] \equiv \bigcap_{i \in I} [\underline{p}^i(s), \overline{p}^i(s)] \neq \emptyset$. No arbitrage and Proposition 1 imply that $\sum_{s \in S} \underline{q}(s) \leq 1 \leq \sum_{s \in S} \overline{q}(s)$. Let $a \in [0,1]$ such that $a \sum_{s \in S} \underline{q}(s) + (1-a) \sum_{s \in S} \overline{q}(s) = 1$. For each $s \in S$, let $p(s) = a\underline{q}(s) + (1-a)\overline{q}(s)$. We then have $\sum_{s \in S} p(s) = 1$, so that p is a probability measure such that $p(s) \in [\underline{q}(s), \overline{q}(s)]$ for all $s \in S$, contradicting that $\bigcap_{i \in I} \overline{\pi}^i(e^i) = \emptyset$.

We therefore have that, for some $i, j \in I$ and $s^* \in S$, $[\underline{p}^i(s^*), \overline{p}^i(s^*)] \bigcap [\underline{p}^j(s^*), \overline{p}^j(s^*)] = \emptyset$. Suppose without loss of generality that $\underline{p}^i(s^*) > \overline{p}^j(s^*)$. Let c be such that $\underline{p}^i(s^*) > c > \overline{p}^j(s^*)$. Define bet f on s^* such that $f(s^*) = 1 - c$ and $f(s) = -c$ for $s \neq s^*$. We then have that $\mathbb{E}_p f = p(s^*)(1-c) - (1-p(s^*))c > 0$ for all $p \in \overline{\pi}^i(e^i)$ and $\mathbb{E}_p(-f) = p(s^*)(-1+c) + (1-p(s^*))c > 0$ for all $p \in \overline{\pi}^j(e^j)$.

Define bet $\{f^k\}_{k \in I}$ such that $f^i(s^*) = f(s^*) - \epsilon$ and $f^i(s) = f(s)$ for $s \neq s^*$, for sufficiently small $\epsilon > 0$ such that the above inequality still holds. For each $k \neq i, j$, let $f^k(s^*) = \frac{\epsilon}{|I \setminus \{i,j\}|}$ and $f^k(s) = 0$ for $s \neq s^*$. Let $f^j = -f$.

By Axiom 4, for all $k \in I$, each $p \in \overline{\pi}^k(e^k)$ has full support on S. Hence, for small enough $\epsilon > 0$ we have, for all $k \in I$, $\mathbb{E}_p f^k > 0$ for all $p \in \overline{\pi}^k(e^k)$ and $\mathbb{E}_p(f^k + e^k) > \mathbb{E}_p e^k$. From the second property of $\pi^k(e^k)$, there exists small enough $\lambda^k > 0$ such that $\lambda^k(e^k + f^k) + (1 - \lambda^k)e^k = \lambda^k f^k + e^k \succ^k e^k$. By taking $\lambda = \min_{k \in I} \lambda^k$, we have that $e^k + \lambda f^k \succ^k e^k$ for all $k \in I$. Because $\sum_{k \in I} \lambda f^k = 0$, $\{\lambda f^k\}_{k \in I}$ is an agreeable bet.

Conversely, suppose there is an agreeable bet $\{f^i\}_{i \in I}$. This means that $\sum_{i \in I} f^i = 0$ and each f^i is of the form $f^i(s^i) = a^i$ and $f^i(s) = b^i$ for $s \neq s^i$, where $a^i \neq b^i$. Moreover, $f^i + e^i \succ^i e^i$ for all $i \in I$. From the first property of $\pi^i(e^i)$ and Axiom 4, we have that $\mathbb{E}_p(e^i + f^i) > \mathbb{E}_p e^i$ for all $p \in \pi^i(e^i)$, all $i \in I$. In the proof of Proposition 1, we show that the same inequality $\mathbb{E}_p f^i > 0$ holds for all $p \in \overline{\pi}^i(e^i)$, all $i \in I$. Suppose there is an $\overline{S}$-common belief p. Then, by adding $\mathbb{E}_p f^i > 0$ over all investors we have $0 < \sum_{i \in I} \mathbb{E}_p f^i = \mathbb{E}_p \sum_{i \in I} f^i = 0$, a contradiction. $\qquad\square$

Combining Theorems 1 and 2, we can conclude that if no common belief implies no $\overline{S}$-common belief, then trade in the absence of perceived complexity implies trade in the presence of maximum perceived complexity. As shown by example in Section 2,

this is not true in general, however a sufficient condition (satisfied by several models, such as subjective expected utility, multiplier, mean-variance and smooth ambiguity preferences) is that each investor has a unique subjective belief at the initial allocation, hence his indifference curve has no kinks.

4   Concluding remarks

We characterize the occurence of trade in two environments which are the extreme opposites in terms of the perceived complexity of the payoff structure. The first is the standard setting of no perceived complexity, where each investor is sophisticated enough to be able to understand and trade any combination of the available securities. In the second, each investor has cognitive limitations so that even though he can understand each A-D security separately, he cannot combine two or more and construct a complex trade. In other words, he can only formulate a coarse trade which is measurable with respect to a two-element partition of the state space, consisting of a state and its complement.

Our main question is, can this type of complexity explain market freezes? The answer depends on how investors perceive risk and uncertainty. We first show that for smooth preferences, maximum perceived complexity does not destroy all trading opportunities, meaning that if there is trade with no perceived complexity, there will be trade with maximum perceived complexity. More importantly, the same conclusion would hold even if we were to provide a more detailed but (necessarily) weaker definition of complexity (e.g. generating complexity endogenously through cognitive costs or computational complexity), as long as each investor can trade at least one security. In other words, our result for these preferences that complexity does not impede trade is robust, because it is independent of the particular details of how a complex security is defined. Examples of smooth preferences are subjective expected utility, the smooth variational preferences of Maccheroni et al. (2006) (including, as special cases, the mean-variance preferences of Markowitz (1952) and Tobin (1958) and the multiplier preferences of Hansen and Sargent (2001)), the smooth ambiguity of Klibanoff et al. (2005) and the second-order expected utility of Ergin and Gul (2009).

If preferences are non-smooth, we find that maximum perceived complexity can impede trade. These are preferences for which there are multiple sets of A-D prices that support zero net demand at the initial allocation, because the indifference curve has a kink at the endowment point. Examples are the CEU with convex capacity of Schmeidler (1989), the MEU of Gilboa and Schmeidler (1989), the non-smooth variational

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preferences of Maccheroni et al. (2006), the confidence preferences of Chateauneuf and Faro (2009) and the uncertainty averse preferences of Cerreia-Vioglio et al. (2011). For instance, in the MEU model with a full insurance allocation, each investor's subjective beliefs coincide with his (non unique) priors and his indifference curve has a kink at his endowment. As shown in Figure 1, in that case maximum perceived complexity completely shuts down trade. This means that a policy which simplifies the securities can have a positive welfare effect. An interesting next step would be to examine these welfare effects using a more detailed but weaker definition of complexity. This is relegated to future research.

Our results make heavy use of the following property of convex preferences. If the expectation of security f^i, according to all of i's subjective beliefs at his endowment e^i, is higher than the expectation of e^i, then there exists a convex combination of f^i and e^i that investor i strictly prefers to e^i. This property is also true in the incomplete preferences model of Bewley (1986), which means that are our results apply there as well. The distinctive property of this model is that indifference curves have a kink at each endowment, whereas in some well known models with convex and complete preferences, such as MEU, kinks appear only at a full insurance endowment. Our results then predict that maximum perceived complexity would shut down trade at all allocations in the Bewley (1986) model but only at full insurance allocations in the MEU model.

Finally, our results can be generalized to an incomplete markets setting. We can model an incomplete markets structure by having a partition $\mathcal{P}$ of S. Given event $E \in \mathcal{P}$, a security pays 1 if E occurs and 0 otherwise. A constrained Pareto optimal allocation (i.e. no trade in the absence of complexity) is then characterized by the existence of a $\mathcal{P}$-common belief, meaning that all investors agree on all events in $\mathcal{P}$ according to one of their subjective beliefs. Similarly, no trade in the presence of maximum perceived complexity is characterized by a $\overline{\mathcal{P}}$-common belief, so that there exists a probability measure which is between the maximum and minimum probability assigned to each event $\mathcal{P}$, by all investors.

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